Ginzburg Landau theory of superconductivity at fractal dimensions

Chul Koo Kim*, A. Rakhimov † and Jae Hyung Yee ‡

Institute of Physics and Applied Physics, Yonsei University, Seoul, 120-749, Korea

Abstract

The post Gaussian effective potential in $D = 2+2\varepsilon$ dimensions is evaluated for the Ginzburg-Landau theory of superconductivity. Two and three loop integrals for the post Gaussian correction terms in $D = 2+2\varepsilon$ dimensions are calculated and ε-expansion for these integrals are constructed. In $D = 2+2\varepsilon$ fractal dimensions Ginzburg Landau parameter turned out to be sensitive to ε and the contribution of the post Gaussian term is larger than that for $D = 3$. Adjusting ε to the recent experimental data on $\kappa(T)$ for high -T_c cuprate superconductor $Tl_2Ca_2Ba_2Cu_3O_{10}(T\ell - 2223)$, we found that $\varepsilon = 0.21$ is the best choice for this material. The result clearly shows that, in order to understand high - T_c superconductivity, it is necessary to include the fluctuation contribution as well as the contribution from the dimensionality of the sample. The method gives a theoretical tool to estimate the effective dimensionality of the samples.

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*E-mail: ckkim@phya.yonsei.ac.kr

†E-mail: rakhimov@rakhimov.ccc.uz. Permanant address: Institute of Nuclear Physics, Tashkent, Uzbekistan (CIS)

‡E-mail: jhyee@phya.yonsei.ac.kr

I. INTRODUCTION

The Ginzburg-Landau (GL) theory of superconductivity [1] had been proposed long before the famous BCS microscopic theory of superconductivity was discovered. A few years after the appearance of the BCS theory, Gorkov derived the GL theory from the BCS theory [2]. Since then, the GL theory has remained as a main theoretical model in understanding superconductivity. It is highly relevant for the description of both type -I [3] and type II superconductors, even though the original BCS theory is inadequate to treat both materials. The success of the GL theory in the study of modern problems of superconductivity lies on its universal effective character in which the details of the microscopic model are unimportant.

Even at the level of meanfield approximation (MFA), the GL theory yields significant information such as the penetration depth (ℓ) and the coherence length (ξ) of the superconducting samples. Many unconventional properties of superconductivity connected with the break down of the simple MFA has been studied both analytically [4] and numerically using the GL theory [5]. Particularly, the fluctuations of the gauge field were studied recently by Camarda et. al. [6] and Abreu et. al. [7] in the Gaussian approximation of the field theory. The effective mass parameters of the Gaussian effective potential (GEP), Ω and Δ , were interpreted as inverses of the coherent length $\xi = 1/\Omega$ and of the penetration depth $\ell = 1/\Delta$, respectively.

In our previous paper [8] we have estimated corrections to the Gaussian effective potential for the $U(1)$ scalar electrodynamics, which represents the standard static GL model of superconductivity. Although it has been shown that the correction is significant in $D = 3$ dimensions, it was not large enough to explain the experimental findings. At the same time, we have investigated the role of quasi two dimensionality in the high T_c superconductivity, by calculating the Gaussian effective potential for $D = 2 + 2\varepsilon$. It was found that the dimensional contribution at the Gaussian approximation level gives the correction in the right direction, but is not large enough to explain the experimental data [8]. However, it is known that fluctuation contributions are much larger in lower dimensions. Therefore, it is necessary to investigate whether the post Gaussian correction terms in $D = 2 + 2\varepsilon$ dimensions provide significant contribution to the mean field result, in order to understand the layered structure of the high T_c superconductivity. In the present paper, we study the role of the post Gaussian contributions in $D = 2 + 2\varepsilon$ dimensions by using the method developed in [8].

The paper is organized as follows: in Section II the GL action is introduced and basic equations are derived; in Section III, the theoretical results for $D = 2+2\varepsilon$ will be compared to existing high T_c experimental data, so that the role of fractal dimensions can be discussed. In the Appendix we calculated two and three loop integrals in $D = 2 + 2\varepsilon$ dimensions.

II. BASIC EQUATIONS FOR THE EFFECTIVE MASSES

The Hamiltonian of the model and explicit expressions for the effective potential in Euclidean D-dimensional space were given in [6–8]. Here we bring the main points for convenience. The effective potential, i.e., the free energy density, $V_{\text{eff}} = \mathcal{F}/\mathcal{V}$ is defined as

$$V_{\text{eff}} = -\ln Z \tag{2.1}$$

where the partition function is

$$Z = \int \mathcal{D}\phi \mathcal{D}A_T \exp\{-\int d^D x H + \int d^D x j\phi + (\vec{j}_A \vec{A})\}. \tag{2.2}$$

The Hamiltonian density is given by

$$H = \frac{1}{2}(\vec{\nabla} \times \vec{A})^2 + \frac{1}{2}(\vec{\nabla}\phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda\phi^4 + \frac{1}{2}e^2\phi^2 A^2 + \frac{1}{2\eta}(\vec{\nabla}\vec{A})^2 \tag{2.3}$$

where we have introduced a gauge fixing term with the limit $\eta \to 0$ being taken after the calculations are carried out. Note that, we are using natural units employing ξ_0 (coherence length at zero temperature) and T_c (critical temperature) as the length and the energy scales, respectively, introduced by [9]:

$$\begin{aligned} &m \to m\xi_0^{-1}, \qquad x \to x\xi_0, \\ &e^2 \to e^2\xi_0^{-1}T_c^{-1}, \quad \lambda \to \lambda\xi_0^{-1}T_c^{-1}. \end{aligned} \tag{2.4}$$

Using the method introduced in refs. [8,10,11] one finds following effective potential:

$$V_{\text{eff}} = V_G + \Delta V_G \tag{2.5}$$

where V_G is the Gaussian part:

$$\begin{aligned} V_G = {} & I_1(\Omega) + \frac{1}{2}I_1(\Delta) + \frac{1}{2}m^2\phi_0^2 + \lambda\phi_0^4 + \frac{1}{2}I_0(\Omega)[m^2 - \Omega^2 + 6\lambda I_0(\Omega) + 12\lambda\phi_0^2] \\ & + I_0(\Delta)[-\Delta_0^2 + e^2 I_0(\Omega) + e^2\phi_0^2], \end{aligned} \tag{2.6}$$

and ΔV_G is the correction part:

$$\begin{aligned} \Delta V_G = {} & [-\frac{1}{2}e^4 \mathrm{I}_2(\Delta) - 18\mathrm{I}_2(\Omega)\lambda^2]{\phi_0}^4 + \{-3\lambda \mathrm{I}_2(\Omega)[-\Omega^2 + \mathrm{m}^2 + 2\mathrm{I}_0(\Delta)\mathrm{e}^2 + 12\lambda\mathrm{I}_0(\Omega)] \\ & - e^2\mathrm{I}_2(\Delta)[-\Delta^2 + \mathrm{e}^2\mathrm{I}_0(\Omega)] - 8\lambda^2\mathrm{I}_3(\Omega,\Omega) - \frac{2}{3}\mathrm{e}^4\mathrm{I}_3(\Delta,\Omega)\}{\phi_0}^2 - \frac{1}{8}\mathrm{I}_2(\Omega)[-\Omega^2 + \mathrm{m}^2 + 2\mathrm{I}_0(\Delta)\mathrm{e}^2 \\ & + 12\lambda\mathrm{I}_0(\Omega)]^2 - \frac{1}{2}\mathrm{I}_2(\Delta)[-\Delta^2 + \mathrm{e}^2\mathrm{I}_0(\Omega)]^2 - \frac{1}{12}\mathrm{e}^4\mathrm{I}_4(\Delta,\Omega) - \frac{1}{2}\lambda^2\mathrm{I}_4(\Omega,\Omega). \end{aligned} \tag{2.7}$$

In the above following integrals are introduced:

$$
\begin{aligned}
I_0(M) &= \int \frac{d^Dp}{(2\pi)^D}\frac{1}{(M^2+p^2)}, \quad I_1(M) = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\ln(M^2+p^2),\\
I_2(M) &= \frac{2}{(2\pi)^D}\int \frac{d^Dk}{(k^2+M^2)^2},\\
I_3(M_1,M_2) &= \frac{1}{(2\pi)^{2D}}\int \frac{d^Dkd^Dp}{(k^2+M_1^2)(p^2+M_1^2)((k+p)^2+M_2^2)},
\end{aligned} \tag{2.8}
$$

$$
I_4(M_1,M_2) = \frac{1}{(2\pi)^{3D}}\int \frac{d^Dkd^Dpd^Dq}{(k^2+M_1^2)(p^2+M_1^2)(q^2+M_2^2)}\frac{1}{((k+p+q)^2+M_2^2)}.
$$

For $D = 3-2\varepsilon$, these integrals were calculated in dimensional regularization in ref. [12] and for $D = 2+2\varepsilon$ in the Appendix of the present paper.

The parameters Ω and Δ are determined by the principle of minimal sensitivity (PMS):

$$
\begin{aligned}
\frac{\partial V_{\rm eff}}{\partial\Omega} &= 0.0362\frac{\lambda^2}{\varepsilon^2} + \frac{\lambda}{\varepsilon}[0.075(\bar\Omega^2-m^2) - 0.108\lambda(\ln\frac{\mu^2}{\bar\Omega^2}+1) - 0.911\bar\phi_0^2\lambda\\
&+ 0.145\lambda^2\ln^2\frac{\mu^2}{\bar\Omega^2} + [(0.290+1.823\bar\phi_0^2)\lambda^2 + 0.151\lambda(m^2-\bar\Omega^2)]\ln\frac{\mu^2}{\bar\Omega^2} + 0.064\lambda^2(1+3\bar\phi_0^2)^2\\
&+ (m^2-\bar\Omega^2)[0.039(m^2-\bar\Omega^2) + 0.954\lambda\bar\phi_0^2 0.151\lambda] - 0.108\lambda^2\ln^3\frac{\mu^2}{\bar\Omega^2}\\
&+ [0.113(\bar\Omega^2-m^2)\lambda - \lambda^2(0.326+1.367\bar\phi_0^2)]\ln^2\frac{\mu^2}{\bar\Omega^2} + [(\bar\Omega^2-m^2)(0.954\lambda\bar\phi_0^2\\
&+ 0.227m^2)\lambda - 0.039(\bar\Omega^2-m^2)) - \lambda^2(0.133+5.729\bar\phi_0^4+3.215\bar\phi_0^2)]\ln\frac{\mu^2}{\bar\Omega^2}\\
&- \lambda^2(0.960\bar\phi_0^2+5.72\bar\phi_0^4+0.144) + \varepsilon[(\bar\Omega^2-m^2)(0.954\lambda\bar\phi_0^2 - 0.062\lambda - 0.039(\bar\Omega^2-m^2)] + O(\varepsilon^2) = 0 \quad ;
\end{aligned} \tag{2.9}
$$

$$
\begin{aligned}
\frac{\partial V_{\rm eff}}{\partial\Delta} &= \frac{(0.334\bar\Omega^2-0.319\lambda)}{\varepsilon} + (0.639\lambda-0.334\bar\Omega^2)\ln\frac{\mu^2}{\bar\Omega^2} + (0.319\lambda-0.334\bar\Omega^2)\ln\frac{\mu^2}{\bar\Delta^2} + (4.015\lambda-4.205\bar\Omega^2)\bar\phi_0^2\\
&- 1.003\bar\Omega^2 + 0.334m^2 + \varepsilon\{(0.167\bar\Omega^2-0.479\lambda)\ln^2\frac{\mu^2}{\bar\Omega^2}\\
&+ [(0.334\bar\Omega^2-0.639\lambda)\ln\frac{\mu^2}{\bar\Delta^2} + 1.003\bar\Omega^2 - 0.334m^2 - 4.015\lambda\bar\phi_0^2]\ln\frac{\mu^2}{\bar\Omega^2}\\
&+ [(4.205\bar\Omega^2-4.015\lambda)\bar\phi_0^2 + 0.334\bar\Omega^2 - 0.334m^2]\ln\frac{\mu^2}{\bar\Delta^2} - 0.262\lambda - 4.943\bar\Omega^2\bar\Delta^2\\
&+ 8.410\bar\Omega^2\bar\phi_0^2 + 0.275\bar\Omega^2\} + O(\varepsilon^2) = 0,
\end{aligned} \tag{2.10}
$$

where we denote optimal values of Ω and Δ by $\bar\Omega$ and $\bar\Delta$, respectively, and $\bar\phi_0$ is a stationary point defined from the equation:

$$
\begin{aligned}
\frac{\partial V_{\rm eff}}{\partial\phi_0} &= \frac{\lambda\bar\Omega^2\bar\Delta^2(0.456\lambda-0.477\bar\Omega^2)}{\varepsilon} + (0.477\bar\Omega^2-0.911\lambda)\ln\frac{\mu^2}{\bar\Omega^2} - (5.729\lambda-2\bar\Omega^2)\bar\phi_0^2\\
&+ 0.477(\bar\Omega^2-m^2) - 0.119\lambda + \frac{\bar\Omega^2m^2}{2\lambda} + \{[0.683\lambda-0.238\bar\Omega^2]\ln^2\frac{\mu^2}{\bar\Omega^2}\\
&+ [5.729\lambda\bar\phi_0^2 + 0.477(m^2-\bar\Omega^2) + 0.239\lambda]\ln\frac{\mu^2}{\bar\Omega^2} + 0.240\lambda - 0.392\bar\Omega^2\}\varepsilon + O(\varepsilon^2) = 0.
\end{aligned} \tag{2.11}
$$

In the equations (2.9) - (2.11) we have used ε expansion of the loop integrals explicitly and numerical values of ξ_0, T_c and e. For the cuprate $Tl_2Ca_2Ba_2Cu_3O_{10}(T\ell-2223)$ these values are

$$\xi_0 = 1.36nm, \quad T_c = 121.5K, \quad e^2 = 16\pi\alpha k_B T_c \xi_0/\hbar c = 0.0000264. \tag{2.12}$$

III. RESULTS AND DISCUSSIONS

The solutions of the Eqs. (2.9) - (2.11) are related to the experimentally measured GL parameter κ as $\kappa = \ell/\xi = \bar{\Omega}/\bar{\Delta}$. We make an attempt to reproduce recent experimental data on $\kappa(T)$ [13] for high -T_c cuprate superconductor $Tl_2Ca_2Ba_2Cu_3O_{10}(T\ell-2223)$.

For this purpose, we adopt usual linear T dependence of parametrization of m and λ as:

$$m^2 = m_0^2(1-\tau) + \tau m_c^2, \quad \lambda = \lambda_0(1-\tau) + \tau\lambda_c, \quad \tau = T/T_c, \tag{3.1}$$

and calculate κ by solving nonlinear equations (2.9) - (2.11). Due to the parametrization (3.1), the model has in general six input parameters: m_0^2, λ_0, m_c^2, λ_c, ξ_0 (coherent length) and T_c (the critical temperature). The experimental values for the cuprate $T\ell - 2223$ are $\xi_0 = 1.36nm$ and $T_c = 121.5K$. To determine other four parameters we used the following strategy. For each given ε, the parameters m_0^2 and λ_0 are fitted to the experimental values of ξ and ℓ at zero temperature: $\xi_0 = 1.36nm$, $\ell_0 = 163nm$. In dimensionless units, (2.4), we have $\bar{\Omega}_0 = \bar{\Omega}(\tau = 0) = 1$ and $\bar{\Delta}_0 = \bar{\Delta}(\tau = 0) = \xi_0/l_0 = 0.0083$ which are used to calculate m_0^2 and λ_0 from the coupled equations (2.9) - (2.11). This procedure gives the ε dependence of m_0^2 which is presented in Fig. 1 (solid line). As in the case of the Gaussian approximation [8], m^2 remains positive only for very small values of ε, although nonlinearity produces several $m^2 = 0$ solutions in this case. We believe that this smallness again indicates the reliability of the present post Gaussian approximation method.

The parameters m_c^2 and λ_c are fixed in the similar way for each given ε. Actually the quantum fluctuations shift m_c^2 from its zero value given by MFA. On the other hand, the exact experimental values of m_c^2 and λ_c are unknown, since the GL parameter at $T = T_c$ is poorly determined. For this reason, we used the experimental values of ξ_c and ℓ_c at very close points to the critical temperature, $\tau_c = 0.98$ which corresponds to $\bar{\Omega}_c = \bar{\Omega}(\tau_c) = 1/\xi_c = 0.128$ and $\bar{\Delta}_c = \bar{\Delta}(\tau_c) = 1/\ell_c = 0.0043$ ($\kappa_c = 29.6$). Then solving the equations (2.9) - (2.11) numerically with respect to m_c and λ_c, we fix these parameters.

After having fixed the input parameters, the temperature dependence of $\bar{\Omega}(\tau)$, $\bar{\Delta}(\tau)$ as well as the GL parameter $\kappa = \bar{\Omega}(\tau)/\bar{\Delta}(\tau)$ are established by solving the gap equations numerically for each ε. Clearly, the solutions of nonlinear gap equations are not unique. In numerical calculations we separated the physical solutions by observing the sign of $\bar{\phi}_0^2$, which

should be positive and that the effective potential at the stationary point $V_{\rm eff}(\bar{\phi}_0)$ should has a real minimum at this point. For $\varepsilon \geq 0.1$, there is a possibility to adjust ε to the recent experimental data on $\kappa(T)$ [13] for high -T_c cuprate superconductor $Tl_2Ca_2Ba_2Cu_3O_{10}(T\ell - 2223)$. Our calculations show that, the best choice of ε is found to be $\varepsilon = 0.21$. The appropriate $\kappa(\tau)$ is presented in Fig. 2 (solid line). The dashed line in this figure shows $\kappa(\tau)$ for $D = 3$. This fitting process allows us to get an estimation on the effective dimensionality of the high - T_c superconducting materials.

IV. SUMMARY

In the present paper, we have carried out two and three loop calculations on the Ginzburg-Landau effective potential beyond the Gaussian approximation for $D = 2 + 2\varepsilon$ fractal dimensions. The result clearly shows that the higher order corrections are substantially large to explain the existing experimental data.

This result strongly suggests that in order to explain the experimental data on high -T_c superconductivity it is necessary to include the fluctuation contribution as well as the contribution from the quasi two dimensionality. We have found that the GL parameter is rather sensitive to ε when the loop corrections to the simple Gaussian approximation are taken into account. The optimal value of ε for the cuprate $(T\ell - 2223)$ is $\varepsilon = 0.21$. It would be interesting to estimate optimal ε in fractal dimensions for other cuprates also.

It is to be noted that we have calculated two and three loop integrals in $D = 2 + 2\varepsilon$ dimensions using the method of dimensional regularization.

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Appendix

EXPLICIT EXPRESSION FOR THE LOOP INTEGRALS IN $D = 2 + 2\varepsilon$ DIMENSION.

Here, we consider the loop integrals defined in Eqs. (2.8) in $D = 2 + 2\varepsilon$ dimensions. In dimensional regularization the integrals $I_0(m)$, $I_1(m)$ and $I_2(m)$ can be easily calculated in momentum space:

$$I_0(m) = \int \frac{d^Dp}{(2\pi)^D}\frac{1}{(m^2+p^2)} = (\frac{e^\gamma\mu^2}{4\pi})^{-\varepsilon}\frac{2\pi^{D/2}}{\Gamma(D/2)(2\pi)^D}\int_0^\infty \frac{k^{D-1}dk}{(k^2+m^2)}$$
$$= (\frac{e^\gamma x}{4\pi})^{-\varepsilon}\frac{\Gamma(-\varepsilon)}{(4\pi)^{1+\varepsilon}} = -\frac{1}{4\pi}\{\frac{1}{\varepsilon} - \ln(x) + \varepsilon[\frac{\pi^2}{12} + \frac{\ln^2(x)}{2}] + O(\varepsilon^2)\}$$
$$I_1(m) = \frac{1}{2}\int \frac{d^Dp}{(2\pi)^D}\ln(k^2+m^2) = -\frac{m^2}{8\pi}\{\frac{1}{\varepsilon} - 1 - \ln(x) + \ \varepsilon[\ln(x) + \frac{\pi^2}{12} + 1$$
$$+ \frac{\ln^2(x)}{2}] + O(\varepsilon^2)\}$$
$$I_2(m) = 2\int \frac{d^Dp}{(2\pi)^D(k^2+m^2)^2} = \frac{1}{2m^2\pi}\{1 - \varepsilon\ln(x) + O(\varepsilon^2)\}, \tag{A.1}$$

with $x = \mu^2/m^2$.

Two and three loop integrals (I_3 and I_4) require a little more effort. It is more convenient to evaluate them in coordinate space rather than in momentum space, since

$$I_3(M_1, M_2) = \frac{1}{(2\pi)^{2D}}\int \frac{d^Dkd^Dp}{(k^2+M_1^2)(p^2+M_1^2)((k+p)^2+M_2^2)} = (\frac{e^\gamma\mu^2}{4\pi})^\varepsilon \int d^Dr G_1^2(r)G_2(r)$$
$$I_4(M_1, M_2) = \frac{1}{(2\pi)^{3D}}\int \frac{d^Dkd^Dpd^Dq}{(k^2+M_1^2)(p^2+M_1^2)(q^2+M_2^2)}\frac{1}{((k+p+q)^2+M_2^2)} \tag{A.2}$$
$$= (\frac{e^\gamma\mu^2}{4\pi})^\varepsilon \int d^Dr G_1^2(r)G_2^2(r),$$

where $G_n(r)$ is the Fourier transform of the propagator $1/(k^2+M_n^2)$ $(n = 1, 2)$:

$$G(r) = \int \frac{d^Dk e^{ikr}}{(2\pi)^D(k^2+m^2)} = \frac{(2\pi)^{-D/2}m^{D-2}}{(mr)^{D/2-1}}K_{D/2-1}(mr) \tag{A.3}$$

and $K_\nu(z)$ is the modified Bessel function. In dimensional regularization, for $D = 2+2\varepsilon$, $G(r)$ is simplified as

$$G(r) = (\frac{e^\gamma x}{2})^{-\varepsilon}\frac{(mr)^{-\varepsilon}}{2\pi}K_\varepsilon(mr). \tag{A.4}$$

Now, substituting (A.4) into (A.3) one notices that unlike in the case of $D = 3-2\varepsilon$, in $D = 2+2\varepsilon$ dimensions there is no singularity at small r and hence the integration can be performed directly from $r = 0$ to $r = \infty$ without splitting radial integration into two regions with small r and large r.

The case with equal masses, $M_1 = M_2 \equiv m$, can be done analytically:

$$I_N(m) = \frac{2^{-N\varepsilon}(e^\gamma x/4)^{\varepsilon(1-N)}}{(2\pi)^{N-1}m^2\Gamma(1+\varepsilon)}\tilde{I}_N(\varepsilon), \qquad \tilde{I}_N(\varepsilon) = \int_0^\infty t^{1+2\varepsilon}[t^{-\varepsilon}K_\varepsilon(t)]^N dt \tag{A.5}$$

for $N = 3, 4$, where the integrals $\tilde{I}_3(\varepsilon)$ and $\tilde{I}_4(\varepsilon)$ are expressed in term of the hypergeometric functions:

$$\tilde{I}_3(\varepsilon) = \frac{\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{2^{3+\varepsilon}}\{\frac{4^\varepsilon\sqrt{\pi}\Gamma(1-2\varepsilon)}{\Gamma(3/2-\varepsilon)}{}_2F_1[1,1-2\varepsilon;\frac{3}{2}-\varepsilon;\frac{1}{4}]$$

$$-\,2\Gamma(1-\varepsilon)\;{}_2F_1[1,1-\varepsilon;\frac{3}{2};\frac{1}{4}]\},\quad (\varepsilon \leq 0.5);$$

$$\tilde{I}_4(\varepsilon) = \frac{\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{8}\{\frac{\varepsilon\Gamma^2(-\varepsilon)}{4^\varepsilon}{}_3F_2[1,1-\varepsilon,\frac{1}{2}+\varepsilon;\frac{3}{2},1+2\varepsilon;1]$$

$$+\,\frac{2\Gamma^2(-\varepsilon)\varepsilon}{4^\varepsilon(2\varepsilon-1)}\;{}_3F_2[\frac{1}{2},1,1-2\varepsilon;\frac{3}{2}-\varepsilon,1+\varepsilon;1]$$

$$-\,\frac{4^\varepsilon\sqrt{\pi}\Gamma(1-3\varepsilon)\Gamma(1+\varepsilon)\Gamma(-2\varepsilon)}{\Gamma(\frac{3}{2}-2\varepsilon)}{}_2F_1[1-3\varepsilon,\frac{1}{2}-\varepsilon;\frac{3}{2}-2\varepsilon;1]\},\quad (\varepsilon \leq 1/3). \tag{A.6}$$

The method of ref. [14] gives the following ε expansion:

$$\begin{aligned} I_3(m) &= \frac{1}{4\pi^2 m^2}[0.5917+\varepsilon(0.6629-1.1835\ln x)+O(\varepsilon^2)], \\ I_4(m) &= \frac{1}{8\pi^3 m^2}[1.188-\varepsilon(2.759+3.5656\ln x)+O(\varepsilon^2)] \end{aligned} \tag{A.7}$$

which is used in our practical calculations.

The case with nonequal masses is rather complicated and cannot be done analitically in general. However, in the particular case, when $\alpha \equiv M_2/M_1 < 1$ [1] the problem may be overcome by expansion in power series in α. We shall illustrate this approximation for $I_3(M_1, M_2)$ below. Using Eq.s (A.3) and (A.4) one obtains

$$I_3(M_1,M_2) = (\frac{e^\gamma\mu^2}{4\pi})^\varepsilon \int\; G_1^2(r)G_2(r)d^Dr = \frac{1}{4\pi^2 M_1^2\Gamma(\varepsilon+1)}[\frac{x_1x_2\exp(2\gamma)}{2}]^{-\varepsilon}\tilde{I}_3(\alpha,\varepsilon), \tag{A.8}$$

where

$$\tilde{I}_3(\alpha,\varepsilon) = \int_0^\infty tK_\varepsilon^2(t)\;(\alpha t)^{-\varepsilon}K_\varepsilon(\alpha t). \tag{A.9}$$

Now using the series expansion of $K_\nu(z)$

$$\begin{aligned} K_\nu(z) = \frac{\Gamma(\nu)\Gamma(1-\nu)}{2}\{&z^{-\nu}[\frac{2^\nu}{\Gamma(1-\nu)}+\frac{2^{\nu-2}z^2}{\Gamma(2-\nu)}+O(z^4)] \\ &-z^\nu[\frac{2^{-\nu}}{\Gamma(1+\nu)}+\frac{2^{-\nu-2}z^2}{\Gamma(2+\nu)}+O(z^4)]\}, \end{aligned} \tag{A.10}$$

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[1] in the present paper $\alpha = 1/\kappa$ where $\kappa \approx 80$ in the large range of temperature

one may expand the factor $(\alpha t)^{-\varepsilon} K_\varepsilon(\alpha t)$ in power series of α and integrate (A.9) analytically to obtain:

$$\tilde{I}_3(\alpha, \varepsilon) = -\frac{\varepsilon^2 \Gamma(\varepsilon)\Gamma^2(-\varepsilon)}{24(2\varepsilon-1)(2\varepsilon-3)2^\varepsilon}\{(2\varepsilon-1)(2\varepsilon-3)(\alpha^2\varepsilon-\alpha^2-6)$$
$$-3\alpha^{-2\varepsilon}[4\varepsilon-6+\alpha^2(2\varepsilon-1)]+O(\alpha^4)\}. \qquad (A.11)$$

Inserting Eq. (A.11) into the Eq. (A.8) one obtains the following ϵ expansion :

$$I_3(M_1, M_2) = \frac{1}{864\pi^2 M_1^2}\{108(1-\ln\alpha) - 3\alpha^2(6\ln\alpha-5) + \varepsilon[\alpha^2(-18\ln^2\alpha + (36\ln x_1 + 18)\ln\alpha$$
$$-30\ln x_1 + 4) - 216\ln x_1 - 108\ln^2\alpha + 216 + 216\ln x_1 \ln\alpha + O(\varepsilon^2)]\}. \qquad (A.12)$$

Similarly, one may calculate $I_4(M_1, M_2)$ to obtain it's ϵ expansion:

$$I_4(M_1, M_2) = \frac{1}{1728\pi^3 M_1^2}\{4\alpha^2(2+9\ln^2\alpha - 6\ln\alpha) - 108\ln^2\alpha + 190.9588\ln\alpha - 280.5109$$
$$+\varepsilon\,[\alpha^2(72\ln^3\alpha - (60+108\ln x_1)\ln^2\alpha + (72\ln x_1 - 28)\ln\alpha - 24\ln x_1 + 23.4519)$$
$$-360\ln^3\alpha + (547.8351 + 324\ln x_1)\ln^2\alpha - (572.8764\ln x_1 + 337.6413)\ln\alpha$$
$$+841.5330\ln x_1 - 806.1519 + O(\varepsilon^2)]\} \qquad (A.13)$$

where, for simplicity, we used explicit values of constants such as γ, $\zeta(3)$, $\ln(2)$, etc.

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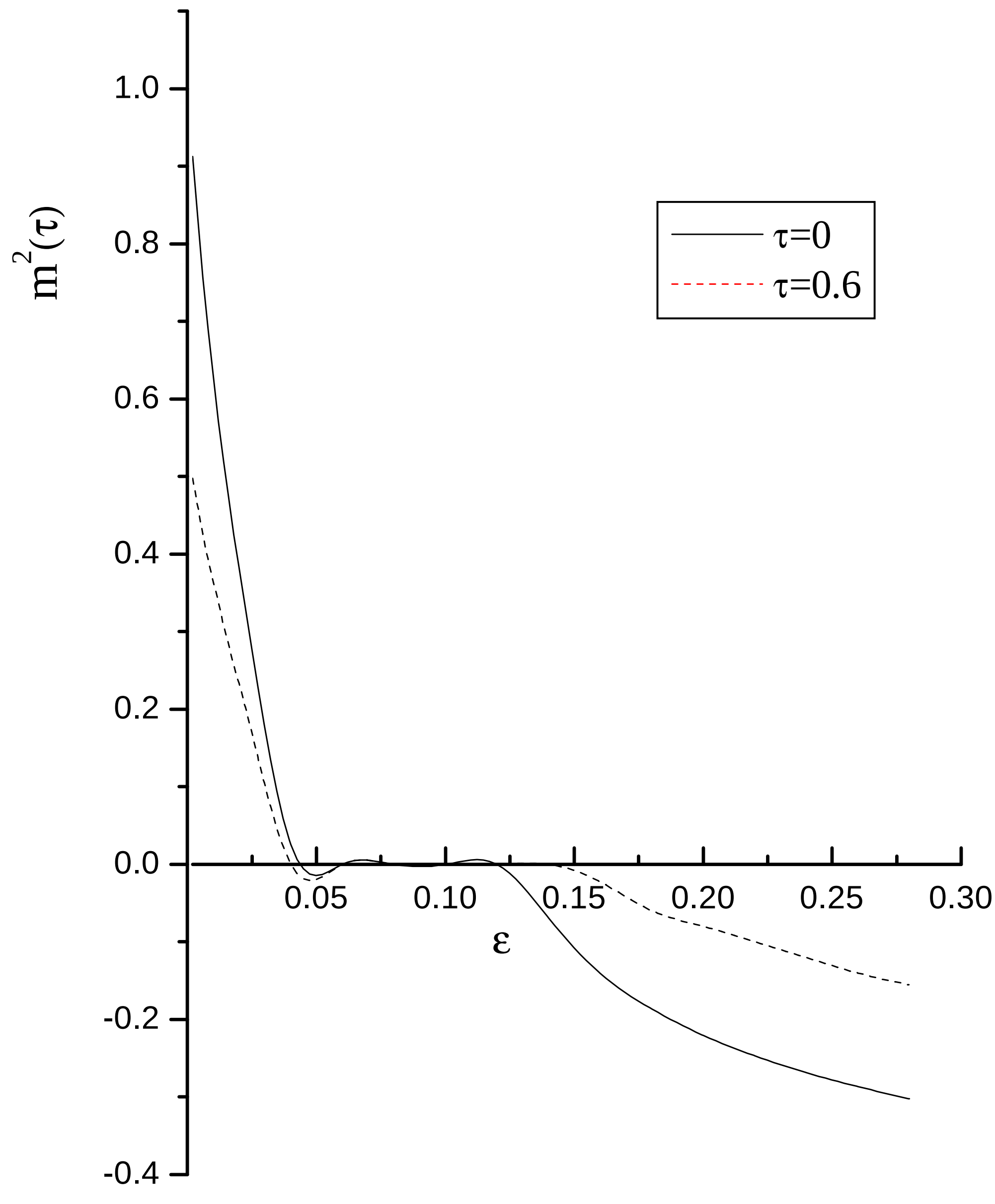

FIG. 1. The parameter m^2 of the GL model v.s. ε in fractal dimension $D = 2 + 2\varepsilon$. The solid and dashed lines are for the temperatures $T = 0$ and $T = 0.6T_c$ respectively.

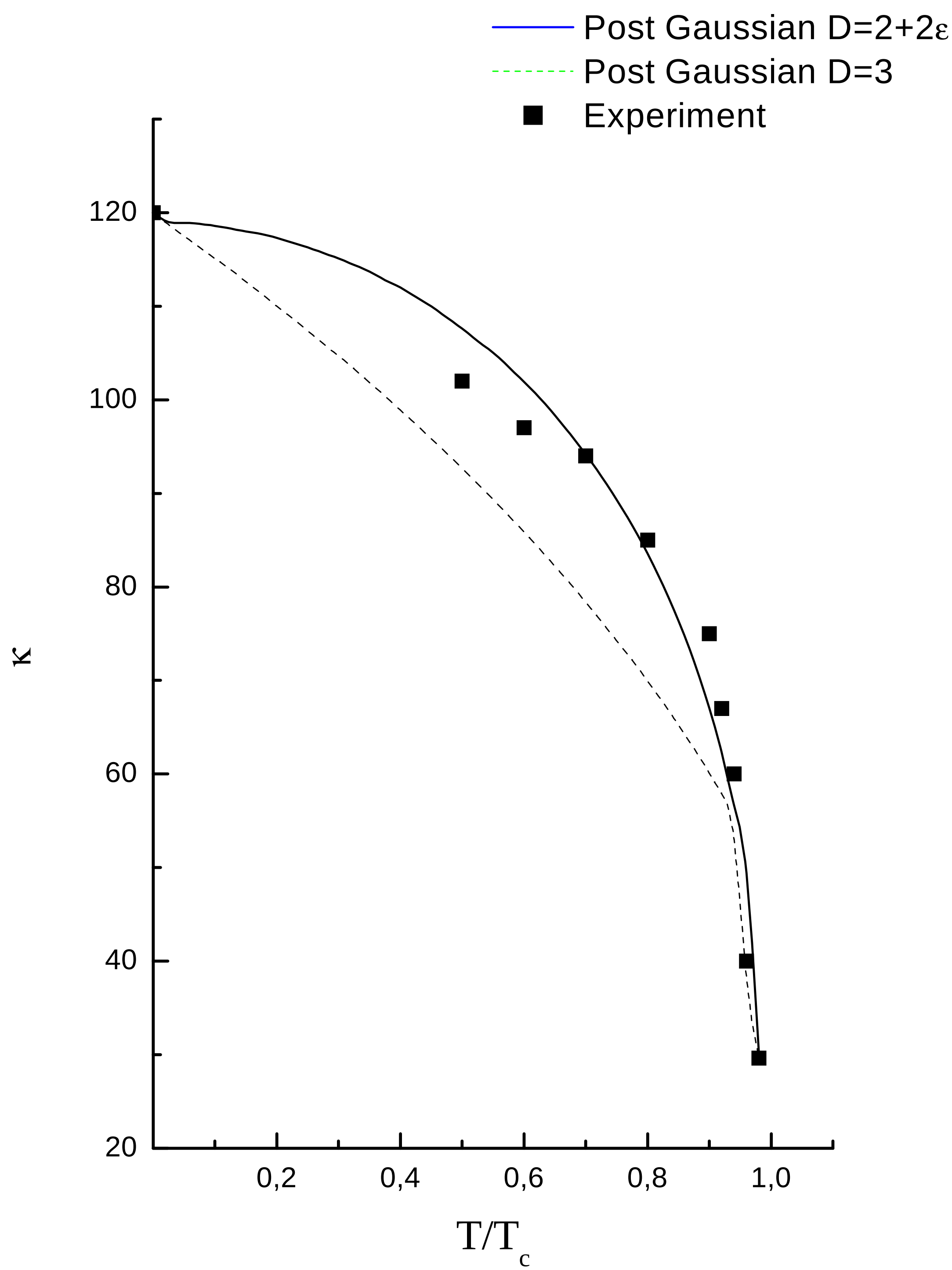

FIG. 2. The GL parameter, κ, in $D = 2 + 2\varepsilon$ (solid line) and in $D = 3$ (dashed line) cases calculated in the PostGaussian approximation.